A Simple Approach to Risk-Adjusted Performance

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Abstract

This note shows that the risk-adjusted performance measure developed by Modigliani and Modigliani [1997] can be derived by using a simple concept from geometry.

INTRODUCTION

Modigliani and Modigliani [1997] derived a risk-adjusted performance (RAP) measure by adjusting the risk of a particular portfolio so that it matches the risk of a market portfolio and then calculate the appropriate return for that portfolio. Unlike Sharpe measure [1966], Treynor measure [1965], and Jensen measure [1968], the unique feature of RAP is that it measures the performance of a portfolio in basis points, the traditional unit to measure return, and hence allow investors to compare the RAP of a portfolio directly with the return of a market portfolio. A high (low) RAP indicates that the portfolio has outperformed (underperformed) the market portfolio. Although Tam [1999] refers to the risk-adjusted performance by Modigliani and Modigliani [1997] as "M-squared," this note deliberately avoids the use of "M-squared" in order not to confuse readers with "M-squared" in Fong and Vasicek [1984] for a risk measurement of an immunized portfolio.

The objective of this note is to simplify the derivation of risk-adjusted performance measure (Equation (4) in Modigliani and Modigliani [1997]). The risk-adjusted performance measure can be derived by using a simple concept from geometry. This simplification will provide readers and investors with an additional insight to the risk-adjusted performance measure.

METHODOLOGY

This note uses the concept of similar triangles from geometry to show the derivation of RAP. From Nowlan and Washburn [1975, p. 266]:
\[ \Delta \text{ABE} \sim \Delta \text{ACD}. \text{ Therefore, } \frac{\text{AB}}{\text{AC}} = \frac{\text{BE}}{\text{CD}} \]

**RESULTS**

In order to avoid crowding out the graph, Figure 1 is a replica of Exhibit 1 in Modigliani and Modigliani [1997] but only for Portfolio 1, \( P_1 \), where the risk of this portfolio \( (\sigma_1) \) is greater than the risk of the market portfolio \( (\sigma_M) \). Although \( P_1 \) offers a higher return than the market portfolio, it also has a higher risk and thus \( r_1 \) is not compatible with \( r_M \). RAP(1) is the risk-adjusted performance of \( P_1 \) after adjusting its risk to match the risk of market portfolio. The concept of similar triangles in geometry is applied to calculate RAP(1). Note that Figure 1 contains two triangles, both triangles are marked by heavy solid lines. Consequently,

\[
\frac{\sigma_M}{\sigma_1} = \frac{x}{r_f - r_1} \tag{1}
\]

\[
x = \frac{\sigma_M (r_1 - r_f)}{\sigma_1} = \frac{\sigma_M}{\sigma_1} (r_1 - r_f)
\]

\[
\text{RAP}(1) = x + r_f \tag{2}
\]

Equation (2) is the same as Equation (4) in Modigliani and Modigliani [1997].
As long as a portfolio has $\sigma_1$ that is greater than $\sigma_M$ and $r_1$ that is higher than $r_f$,
Equation (1) can be used to calculate the risk-adjusted performance of this portfolio.
Applying the information in Table 1 to Equation (1), the RAP for Fidelity Magellan, for example, is:

\[
\frac{7.2}{8.6} = \frac{x}{15.4 - 5.5} \rightarrow x = 8.29
\]

Substituting $x$ into Equation (2), the RAP for Fidelity Magellan is $8.29 + 5.5 = 13.79 \approx 13.8$, a result similar to the RAP for Fidelity Magellan in Modigliani and Modigliani [1997]. If Fidelity Magellan had the same level of risk as the market portfolio, it would had a 13.8% risk-adjusted return, a return that is lower than the return of the market portfolio (14.1%). For this reason, Fidelity Magellan had underperformed the market portfolio.

Figure 2 is also a replica of Exhibit 1 in Modigliani and Modigliani [1997] and contains the analysis for Portfolio 2, $P_2$, where the risk of this portfolio ($\sigma_2$) is less than the risk of the market portfolio ($\sigma_M$). Even tough $P_2$ offers a lower return than the market portfolio, it also has a lower risk and hence $r_2$ is not compatible with $r_M$. $\text{RAP}(2)$ is the risk-adjusted performance of $P_2$ after adjusting its risk to match the risk of market portfolio. The same geometry approach is used to solve for $\text{RAP}(2)$. Also note that Figure 2 contains two triangles, both triangles are marked by heavy solid lines. Consequently,
\[
\frac{\sigma_M - \sigma_2}{\sigma_M} = \frac{x - (r_2 - r_f)}{x}
\]
(3)

\[
x = \frac{\sigma_M (r_2 - r_f)}{\sigma_2} = \frac{\sigma_M}{\sigma_2} (r_2 - r_f)
\]

\[
\text{RAP}(2) = x + r_f
\]
(4)

Equation (4) is also identical to Equation (4) in Modigliani and Modigliani [1997].

For a portfolio with \(\sigma_2\) that is smaller than \(\sigma_M\) and \(r_2\) that is higher than \(r_f\), Equation (3) can be used to calculate the risk-adjusted performance of this portfolio. Applying the information in Table 1 to Equation (3), the RAP for Fidelity Puritan, for instance, is:

\[
\frac{7.2 - 4.7}{7.2} = \frac{x - (12.0 - 5.5)}{x} \rightarrow x = 9.96
\]

Substituting \(x\) into Equation (4), the RAP for Fidelity Puritan is 9.96 + 5.5 = 15.46 and is compared favorably to the RAP for Fidelity Puritan in Modigliani and Modigliani [1997]. If Fidelity Puritan had the same level of risk as the S&P 500, it would had a 15.5% risk-adjusted return, a return that is higher than the return of the S&P 500 (14.1%). Therefore, Fidelity Puritan had outperformed the S&P 500.

**CONCLUSIONS**

This note simplifies the derivation of risk-adjusted performance measure developed by Modigliani and Modigliani [1997]. The risk-adjusted performance measure can be derived by using a simple concept from geometry.
REFERENCES


Figure 1. RAP for $\sigma_1 > \sigma_M$

where: $r =$ return;

$\sigma =$ standard deviation = risk;

$r_f =$ risk-free rate of return;

$P_1 =$ Portfolio 1, with $r_1 =$ return of Portfolio 1 and $\sigma_1 =$ risk of Portfolio 1;

$P_M =$ market portfolio, with $r_M =$ return of market portfolio and $\sigma_M =$ risk of market portfolio; and

$\text{RAP}(1) =$ risk-adjusted performance of Portfolio 1.
Figure 2. RAP for $\sigma_2 < \sigma_M$

where: $P_2 = \text{Portfolio 2}$, with $r_2 = \text{return of Portfolio 2}$ and $\sigma_2 = \text{risk of Portfolio 2}$; and

$\text{RAP}(2) = \text{risk-adjusted performance of Portfolio 2}$.
### Table 1. RAP: Analysis of Selected Mutual Funds

<table>
<thead>
<tr>
<th>Mutual Funds (in order of total return)</th>
<th>Average Quarterly Total Return (at annual rate)</th>
<th>Quarterly Standard Deviation</th>
<th>Quarterly RAP (at annual rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>14.1</td>
<td>7.2</td>
<td>14.1</td>
</tr>
<tr>
<td>AIM Constellation</td>
<td>19.7</td>
<td>12.3</td>
<td>13.9</td>
</tr>
<tr>
<td>20th Century Vista Investors</td>
<td>16.7</td>
<td>14.0</td>
<td>11.3</td>
</tr>
<tr>
<td>T. Rowe Price New Horizon</td>
<td>16.0</td>
<td>11.3</td>
<td>12.2</td>
</tr>
<tr>
<td>Fidelity Magellan</td>
<td>15.4</td>
<td>8.6</td>
<td>13.8</td>
</tr>
<tr>
<td>Vanguard Windsor</td>
<td>13.0</td>
<td>7.5</td>
<td>12.7</td>
</tr>
<tr>
<td>Fidelity Puritan</td>
<td>12.0</td>
<td>4.7</td>
<td>15.4</td>
</tr>
<tr>
<td>Income Fund of America</td>
<td>11.3</td>
<td>4.0</td>
<td>15.9</td>
</tr>
<tr>
<td>T-Bill</td>
<td>5.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Modigliani and Modigliani [1997], p. 50.